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NOTES ON GREATEST COMMON DIVISOR AND LEAST COMMON MULTIPLE OF INTEGERS.

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I. Properties of Quotients.

Let $q_1, q_2, \ldots, q_n$ be the respective quotients obtained by dividing the positive integers $a_1, a_2, \ldots, a_n$ by their greatest common divisor D; and $q'_1, q'_2, \ldots, q'^n$ the respective quotients obtained by dividing L, the lowest common multiple of the integers $a_1, a_2, \ldots, a_n$, by the integers, in turn. Hence

$$q_1 = a_1 \div D;\ q_2 = a_2 \div D;\ \ldots;\ q_n = a_n \div D \ldots (1);$$
$$q'_1 = L \div a_1;\ q'_2 = L \div a_2;\ \ldots;\ q'_n = L \div a_n \ldots (2).$$

The two sets of quotients possess the following properties:

1. Neither set of quotients has a factor common to all the numbers in the set. For, otherwise, D would not be the greatest common divisor nor L the least common multiple of the numbers $a_1, a_2, \ldots, a_n$.

2. $q_1 q'_1 = q_2 q'_2 = \ldots = q_n q'_n = L \div D$.

This result is obtained by multiplying, member by member, the cor-

responding equalities in (1) and (2). Furthermore, by suitably combining all the equalities in (1) and (2), we get

3. $(q_1 q_2 \ldots q_n)(q'_1 q'_2 \ldots q'_n) = L^n \div D^n$; and,

4. $(a_1 a_2 \ldots a_n)^2 = L^n D^n (q_1 q_2 \ldots q_n) \div (q'_1 q'_2 \ldots q'_n)$.

From the second property stated above it is easily seen that

5. $L \div D$ divided in order by either set of quotients gives in corresponding order the other set.

From the first and fifth properties, it follows that

6. The least common multiple of each set of quotients is $L \div D$.

II. Relation of Integers to their Greatest Common Divisor and Least Common Multiple.

1. As is well known, the product of two positive integers, a_1 and a_2, is equal to the product of their greatest common divisor D and least common multiple L:

$$a_1 a_2 = DL.$$

The general relation may be stated as follows:

$$DL^{n-1} \overline{\overline{>}}\, a_1 a_2 \ldots a_n \overline{\overline{>}}\, D^{n-1} L \ldots \text{(A)}.$$

Proof. From (2) of I, we get

$$L^n = (a_1 a_2 \ldots a_n)(q'_1 q'_2 \ldots q'_n).$$

But, as previously shown, the least common multiple of $q'_1, q'_2, \ldots, q'_n$ is $L \div D$. Hence, $q'_1 q'_2 \ldots q'_n \overline{\overline{>}}\, L \div D$. Therefore, $L^n \overline{\overline{>}}\, (a_1 a_2 \ldots a_n) L \div D$.

$\therefore DL^{n-1} \overline{\overline{>}}\, a_1 a_2 \ldots a_n$.

Similarly, starting with (1) of I, it can be shown that

$$a_1 a_2 \ldots a_n \overline{\overline{>}}\, D^{n-1} L.$$

2. Another form of the general case is as follows:

$$\prod {}_kD_i \prod {}_kL_i^{k-1} \overline{\overline{>}}\, (a_1 a_2 \ldots a_n)^{\frac{(n-1)!}{(n-k)!(k-1)!}} \overline{\overline{>}}\, \prod {}_kD_i^{k-1} \prod {}_kL_i, \ldots \text{(B)},$$

in which ${}_kD_i$ and ${}_kL_i$ represent, respectively, the greatest common divisors and least common multiples of the n integers taken k at a time, i thus being

equal to 1, 2, ..., $\frac{n!}{(n-k)!k!}$.

The proof of this relation follows easily from (A), which is true for each set of k integers selected from the entire set of n integers. There will evidently be, in all, $\frac{n!}{(n-k)!k!}$ such relations. Multiplying together these relations, member by member, and noticing that in the middle member each of the n integers will occur $\frac{(n-1)!}{(n-k)!(k-1)!}$ times, the desired result is obtained.

3. If in (B) we set $k=2$, we obtain

$$(a_1 a_2 \ldots a_n)^{n-1} = \prod {}_2D_i \prod {}_2L_i, \quad i=1, 2, \ldots, \tfrac{1}{2}n(n-1),$$

which is probably the most general equality relation existing between integers, and greatest common divisors and least common multiples.

If, finally, $n=2$, we arrive at the well-known relation

$$a_1 a_2 = DL.$$

A POINT'S VISIT TO THE LINEAR CONTINUUM.

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Once upon a time a point, XYZ, in three-dimensional space, decided to visit the set of points on a certain straight line. He had a sense of his superiority over his less fortunate fellow-beings who were compelled to lead a one-dimensional existence, but his motiqe for visiting them might be called philanthropic for he had a real desire to find out the relations existing between them and to enlighten them, if possible, concerning a higher existence.

As XYZ approached the line he was surprised to find that its residents were not living happily together; and he soon learned that they were capable of the same petty dissensions and jealousies which he had supposed were possible only in the enlightened set to which he belonged. He found that none of them was on speaking terms with his next-door neighbor — in fact, that he did not know who his next-door neighbor was. The society was divided up into political parties, religious denominations, and exclusiqe cliques, which spent most of their time quarrelling with one another, and did not seem to realize that, taken all together, they formed one Grand Continuum.